The β^+ Decay of ^{234}Np and Other Isospin-Forbidden $0^+ \to 0^+$ Fermi Transitions

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Although experimental values of the Fermi nuclear matrix elements vary widely from about 1×10^{-3} to 40×10^{-3} for isospin-forbidden $0^+ \to 0^+$ β transitions, theoretical calculations using the Coulomb potential and Nilsson wave functions yielded values of M_F in reasonably good agreement, except that of ^{234}Np. However, our calculation of M_F for this decay as a function of the deformation parameter β yielded a value of M_F in good agreement with experiment for values of β between 0.1 and 0.2.

Introduction

Recently, much experimental work [1] has been done in isospin-forbidden Fermi transitions which can be classified into two types: $0^+ \to 0^+$ and $J^\pi \to J^\pi$ $(J \neq 0)$ transitions. We are not aware of the existence of any $0^- \to 0^-$ Fermi transitions. In this paper we are only concerned with isospin-forbidden $0^+ \to 0^+$ Fermi transitions where the Fermi nuclear matrix element M_F can be determined from a measurement of the log ft value alone [1]

$$|M_F| = \sqrt{\frac{2\,(3088.6 \pm 2.1)\ \text{sec}}{ft\ \text{sec}}}\ .$$

According to the conserved-vector-current (CVC) theory of Feynman and Gell-Mann [2], if there are no charge-dependent effects, isospin is a good quantum number and the Fermi nuclear matrix elements for β-transitions with $\Delta J = 0$, $\Delta T = \pm 1$ should be zero. The non-vanishing values of M_F could arise from isospin impurities due to charge-dependent effects or from second forbidden corrections to the β matrix element. However, second forbidden contributions are negligible in most cases [3]. Therefore, the observed non-vanishing values of M_F for $\Delta T = \pm 1$ β-decays have to be ascribed to isospin impurities due to charge-dependent forces.

All charge-dependent effects are believed to be electromagnetic in origin [4]. Direct charge-dependent effects arise from the Coulomb potential, the magnetic interaction, the neutron-proton mass difference, the vacuum polarization and the finite charge and magnetic moment distributions of the nucleons. Of course, the main contribution comes from the Coulomb interaction while others can be neglected [4, 5]. Indirect charge-dependent effects, whose contributions to M_F could be as large as those from the Coulomb potential [6, 7] will not be considered here.

Calculation and Results

The significant experimental discovery [8] of resonance peaks of very narrow widths in the study of (p, n) reactions implies that the nuclear matrix elements of Fermi transitions from the target nucleus would be exhausted by the analogue state. With reference to Figure 1, the Fermi nuclear matrix element for either a β^- or a β^+ decay is then given by:

$$\begin{aligned} M_F &= \langle f \,|\, T_\pm \,|\, i \rangle \\ &= \alpha_T \sqrt{(T - T_z)\,(T + T_z + 1)} \\ &= \alpha_T \sqrt{2T} \quad \text{for known } 0^+ \to 0^+ \text{ decays}, \end{aligned}$$

where α_T is the admixture amplitude of the state $|A\rangle$, which is the analogue of the parent state $|P\rangle$, into the anti-analogue state $|T_<\rangle$, and according to

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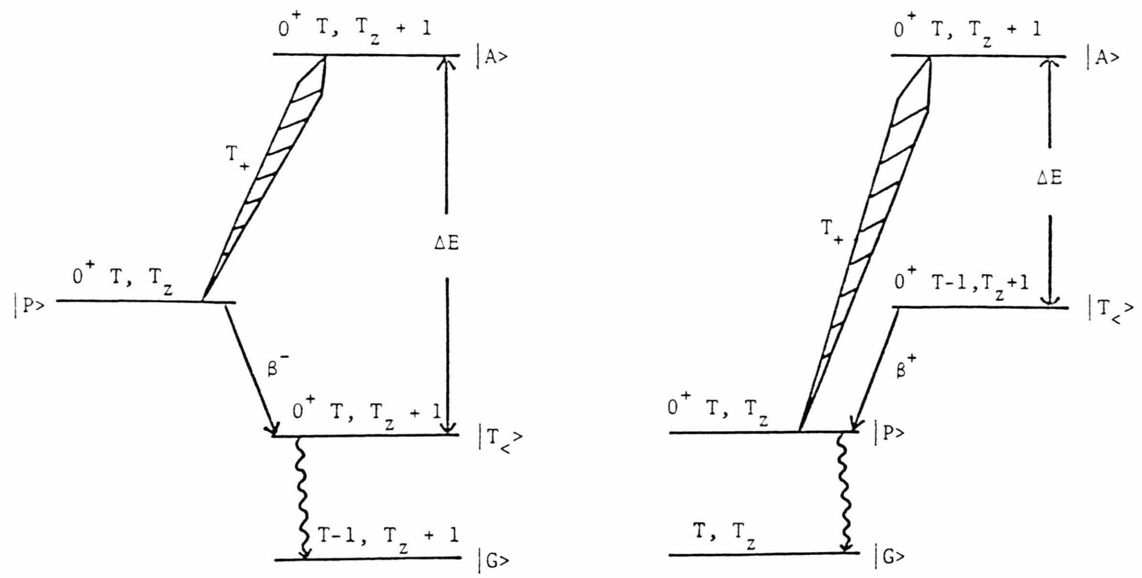

Fig. 1. Decay schemes for isospin $0^+ \rightarrow 0^+$ transitions.

Table 1. Experimental and theoretical values of M_F.

Decay	Log. ft	Exp. $\lvert M_F \rvert \times 10^3$	Theor. $\lvert M_F \rvert \times 10^3$	Reference
$^{64}Ga \xrightarrow{\beta+} {}^{64}Zn$	6.52 ± 0.02	43.4 ± 1.1	71.2	Yap and Tee [10]
$^{66}Ge \xrightarrow{\beta+} {}^{66}Ga$	> 6.59	< 39.5	33.5	Yap and Saw [11]
$^{66}Ga \xrightarrow{\beta+} {}^{66}Zn$	7.90 ± 0.05	8.8 ± 0.6	9 at $\beta \approx 0.25$	Yap and Saw [12]
$^{156}Eu \xrightarrow{\beta-} {}^{156}Gd$	9.82 ± 0.05	0.97 ± 0.06	1.5 $\sim$2 at $\beta = 0.3$	Damgard [3] Saw and Yap [13]
$^{170}Lu \xrightarrow{\beta+} {}^{170}Yb$	9.79 ± 0.20	1.0 ± 0.3	1.0	Damgard [3]
$^{188}W \xrightarrow{\beta-} {}^{188}Re$	10.00 ± 0.20	0.79 ± 0.20	0.72 to 0.93	Shera et al. [14]
$^{234}Np \xrightarrow{\beta+} {}^{234}U$	8.39 ± 0.20	5.0 ± 1.3	25 6 at $\beta = 0.15$	Damgard [3] present work

first-order perturbation theory, it is given by:

$$\alpha_T = -\frac{\langle T_< | V_{CD} | A \rangle}{\Delta E},$$

where V_{CD} is the charge-dependent potential in the nuclear Hamiltonian.

Table 1 gives the complete list of isospin-forbidden $0^+ \rightarrow 0^+$ transitions, together with the values of M_F deduced from experimental results [1] and from theoretical calculations using the Coulomb potential and Nilsson wave functions. The one-body spheroidal Coulomb potential V_{CD} may be written as:

$$V_{CD} = \frac{(Z-1)\,e^2}{R}\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2\right] + a\left(\frac{r}{R}\right)^2 Y_{20} \quad \text{for } r < R$$

$$= \frac{(Z-1)\,e^2}{r} + a\left(\frac{R}{r}\right)^3 Y_{20} \quad \text{for } r > R,$$

where R is the radius of the nucleus and a is related to Bohr's deformation parameter β by:

$$a = \frac{5}{3} \beta \frac{(Z-1)}{R} e^2 .$$

It is obvious from Table 1 that the agreement between theory and experiment is reasonably good except for the β^+-decay of ^{234}Np which we shall now investigate in greater detail. Our calculations show that theoretical values of M_F depend quite sensitively on the deformation parameter β. This sensitivity is due to the fact that at different deformations the expansion coefficients appearing in the Nilsson wave functions are quite different, resulting in quite different values for M_F.

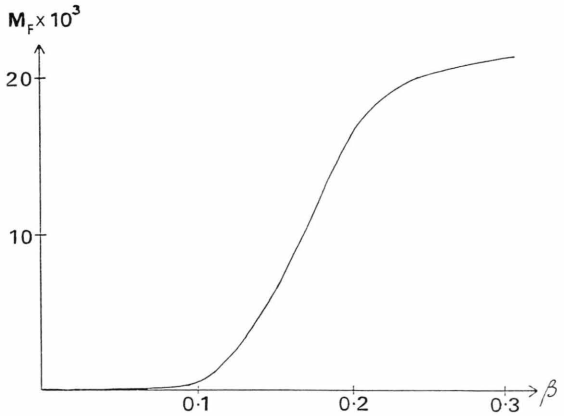

Fig. 2. Variation of M_F with deformation parameter for ^{234}Np.

Table 2. Values of $\Omega^{\pi}[Nn_z\Lambda]$ for various values of β.

β	0.1	0.2	0.3
$\Omega_1^{\pi}[N_1 n_{z1} \Lambda_1]$	$\frac{5^-}{2}$ [523]	$\frac{5^+}{2}$ [642]	$\frac{5^-}{2}$ [523]
$\Omega_2^{\pi}[N_2 n_{z2} \Lambda_2]$	$\frac{5^-}{2}$ [752]	$\frac{5^+}{2}$ [633]	$\frac{5^-}{2}$ [503]

On taking the appropriate configuration assignments for various values of β as given by Table 2, the Fermi nuclear matrix element reduces to:

$$M_F = \frac{\sqrt{2}\,\langle \Omega_1^{\pi}[N_1\, n_{z1}\, \Lambda_1] |\, V_{CD}\, | \Omega_2^{\pi}[N_2\, n_{z2}\, \Lambda_2] \rangle}{\Delta E} .$$

The results of our calculation for various values of the deformation parameter β are presented in Figure 2. From the figure, we see that for values of the deformation parameter between 0.1 and 0.2 agreement between theory and experiment is reasonably good, bearing in mind that the variation of M_F against β is maximum in this region. We therefore could not expect a calculation of this type to give exact agreement. There is no experimental value of β for ^{234}Np but for the daughter nucleus ^{234}U the value of β is [9] 0.22. Is is noted that nuclei around mass 234 have roughly this value. It is noted that our value of M_F agrees with that of Damgard for the larger value of β which he used.

Conclusions

It is well known that experimental values of M_F for isospin-forbidden β transitions are generally very small with only three exceptions. Two of these are isospin-forbidden $0^+ \rightarrow 0^+$ β-decays, thus making the values of M_F vary widely from $\sim 1 \times 10^{-3}$ to $\sim 40 \times 10^{-3}$ for such decays. In spite of this, theoretical calculations using the Coulomb potential and Nilsson wave functions yielded values of M_F in reasonably good agreement with experimental results for all known isospin-forbidden $0^+ \rightarrow 0^+$ β-decays, including that of ^{234}Np.

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